Example 5–23

The *Statistical Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance, and standard deviation of the number of births that would result in twins.

Source: 100% American by Daniel Evan Weiss.

Solution

This is a binomial situation, since a birth can result in either twins or not twins (i.e., two outcomes).

$$\mu = n \cdot p = (8000)(0.02) = 160$$
$$\sigma^2 = n \cdot p \cdot q = (8000)(0.02)(0.98) = 156.8$$
$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{156.8} = 12.5$$

For the sample, the average number of births that would result in twins is 160, the variance is 156.8, or 157, and the standard deviation is 12.5, or 13 if rounded.

Applying the Concepts 5–4

Unsanitary Restaurants

Health officials routinely check sanitary conditions of restaurants. Assume you visit a popular tourist spot and read in the newspaper that in 3 out of every 7 restaurants checked, there were unsatisfactory health conditions found. Assuming you are planning to eat out 10 times while you are there on vacation, answer the following questions.

1. How likely is it that you will eat at three restaurants with unsanitary conditions?
2. How likely is it that you will eat at four or five restaurants with unsanitary conditions?
3. Explain how you would compute the probability of eating in at least one restaurant with unsanitary conditions. Could you use the complement to solve this problem?
4. What is the most likely number to occur in this experiment?
5. How variable will the data be around the most likely number?
6. Is this a binomial distribution?
7. If it is a binomial distribution, does that mean that the likelihood of a success is always 50% since there are only two possible outcomes?

Check your answers by using the following computer-generated table.

Mean = 4.3 **Std. dev. = 1.56557**

X	*P*(*X*)	Cum. Prob.
0	0.00362	0.00362
1	0.02731	0.03093
2	0.09272	0.12365
3	0.18651	0.31016
4	0.24623	0.55639
5	0.22291	0.77930
6	0.14013	0.91943
7	0.06041	0.97983
8	0.01709	0.99692
9	0.00286	0.99979
10	0.00022	1.00000

See page 283 for the answers.

Exercises 5–4

1. Which of the following are binomial experiments or can be reduced to binomial experiments?
   a. Surveying 100 people to determine if they like Sudsy Soap
   b. Tossing a coin 100 times to see how many heads occur
   c. Drawing a card with replacement from a deck and getting a heart
   d. Asking 1000 people which brand of cigarettes they smoke
   e. Testing four different brands of aspirin to see which brands are effective
   f. Testing one brand of aspirin by using 10 people to determine whether it is effective
   g. Asking 100 people if they smoke
   h. Checking 1000 applicants to see whether they were admitted to White Oak College
   i. Surveying 300 prisoners to see how many different crimes they were convicted of
   j. Surveying 300 prisoners to see whether this is their first offense

2. **(ans)** Compute the probability of X successes, using Table B in Appendix C.
   a. $n = 2, p = 0.30, X = 1$
   b. $n = 4, p = 0.60, X = 3$
   c. $n = 5, p = 0.10, X = 0$
   d. $n = 10, p = 0.40, X = 4$
   e. $n = 12, p = 0.90, X = 2$
   f. $n = 15, p = 0.80, X = 12$
   g. $n = 17, p = 0.05, X = 0$
   h. $n = 20, p = 0.50, X = 10$
   i. $n = 16, p = 0.20, X = 3$

3. Compute the probability of X successes, using the binomial formula.
   a. $n = 6, X = 3, p = 0.03$
   b. $n = 4, X = 2, p = 0.18$
   c. $n = 5, X = 3, p = 0.63$
   d. $n = 9, X = 0, p = 0.42$
   e. $n = 10, X = 5, p = 0.37$

For Exercises 4 through 13, assume all variables are binomial. (*Note:* If values are not found in Table B of Appendix C, use the binomial formula.)

4. A burglar alarm system has six fail-safe components. The probability of each failing is 0.05. Find these probabilities.
   a. Exactly three will fail.
   b. Fewer than two will fail.
   c. None will fail.
   d. Compare the answers for parts *a*, *b*, and *c*, and explain why the results are reasonable.

5. A student takes a 20-question, true/false exam and guesses on each question. Find the probability of passing if the lowest passing grade is 15 correct out of 20. Would you consider this event likely to occur? Explain your answer.

6. A student takes a 20-question, multiple-choice exam with five choices for each question and guesses on each question. Find the probability of guessing at least 15 out of 20 correctly. Would you consider this event likely or unlikely to occur? Explain your answer.

7. In a survey, 30% of the people interviewed said that they bought most of their books during the last 3 months of the year (October, November, December). If nine people are selected at random, find the probability that exactly three of these people bought most of their books during October, November, and December.
   Source: USA Snapshot, *USA TODAY.*

8. In a Gallup Survey, 90% of the people interviewed were unaware that maintaining a healthy weight could reduce the risk of stroke. If 15 people are selected at random, find the probability that at least 9 are unaware that maintaining a proper weight could reduce the risk of stroke.
   Source: USA Snapshot, *USA TODAY.*

9. In a survey, three of four students said the courts show "too much concern" for criminals. Find the probability that at most three out of seven randomly selected students will agree with this statement.
   Source: *Harper's Index.*

10. It was found that 60% of American victims of health care fraud are senior citizens. If 10 victims are randomly selected, find the probability that exactly 3 are senior citizens.
    Source: *100% American* by Daniel Evan Weiss.

11. R. H. Bruskin Associates Market Research found that 40% of Americans do not think that having a college education is important to succeed in the business world. If a random sample of five Americans is selected, find these probabilities.
    a. Exactly two people will agree with that statement.
    b. At most three people will agree with that statement.
    c. At least two people will agree with that statement.
    d. Fewer than three people will agree with that statement.
    Source: *100% American* by Daniel Evans Weiss.

12. Find these probabilities for a sample of 20 teenagers if 70% of them had compact disk players by the age of 16.

a. At least 14 had CD players
b. Exactly 9 had CD players
c. More than 17 had CD players
d. Which event, *a*, *b*, or *c*, is most likely to occur? Explain why.

13. If 80% of the people in a community have Internet access from their homes, find these probabilities for a sample of 10 people.

a. At most 6 have Internet access.
b. Exactly 6 have Internet access.
c. At least 6 have Internet access.
d. Which event, *a*, *b*, or *c*, is most likely to occur? Explain why.

14. (ans) Find the mean, variance, and standard deviation for each of the values of n and p when the conditions for the binomial distribution are met.

a. $n = 100, p = 0.75$
b. $n = 300, p = 0.3$
c. $n = 20, p = 0.5$
d. $n = 10, p = 0.8$
e. $n = 1000, p = 0.1$
f. $n = 500, p = 0.25$
g. $n = 50, p = \frac{2}{5}$
h. $n = 36, p = \frac{1}{6}$

15. A study found that 1% of Social Security recipients are too young to vote. If 800 Social Security recipients are randomly selected, find the mean, variance, and standard deviation of the number of recipients who are too young to vote.

Source: *Harper's Index.*

16. Find the mean, variance, and standard deviation for the number of heads when 20 coins are tossed.

17. If 3% of calculators are defective, find the mean, variance, and standard deviation of a lot of 300 calculators.

18. It has been reported that 83% of federal government employees use e-mail. If a sample of 200 federal government employees is selected, find the mean, variance, and standard deviation of the number who use e-mail.

Source: *USA TODAY.*

19. A survey found that 21% of Americans watch fireworks on television on July 4. Find the mean, variance, and standard deviation of the number of individuals who watch fireworks on television on July 4 if a random sample of 1000 Americans is selected.

Source: USA Snapshot, *USA TODAY.*

20. In a restaurant, a study found that 42% of all patrons smoked. If the seating capacity of the restaurant is 80 people, find the mean, variance, and standard deviation of the number of smokers. About how many seats should be available for smoking customers?

21. A survey found that 25% of pet owners had their pets bathed professionally rather than do it themselves. If 18 pet owners are randomly selected, find the probability that exactly five people have their pets bathed professionally.

Source: USA Snapshot, *USA TODAY.*

22. In a survey, 63% of Americans said they own an answering machine. If 14 Americans are selected at random, find the probability that exactly 9 own an answering machine.

Source: USA Snapshot, *USA TODAY.*

23. One out of every three Americans believes that the U.S. government should take "primary responsibility" for eliminating poverty in the United States. If 10 Americans are selected, find the probability that at most 3 will believe that the U.S. government should take primary responsibility for eliminating poverty.

Source: *Harper's Index.*

24. In a survey, 58% of American adults said they had never heard of the Internet. If 20 American adults are selected at random, find the probability that exactly 12 will say they have never heard of the Internet.

Source: *Harper's Index.*

25. In the past year, 13% of businesses have eliminated jobs. If five businesses are selected at random, find the probability that at least three have eliminated jobs during the last year.

Source: *USA TODAY.*

26. Of graduating high school seniors, 14% said that their generation will be remembered for their social concerns. If seven graduating seniors are selected at random, find the probability that either two or three will agree with that statement.

Source: *USA TODAY.*

27. A survey found that 86% of Americans have never been a victim of violent crime. If a sample of 12 Americans is selected at random, find the probability that 10 or more have never been victims of violent crime. Does it seem reasonable that 10 or more have never been victims of violent crime?

Source: *Harper's Index.*